## MEDIATION ANALYSIS: A PRACTITIONER'S GUIDE

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# Mediation Analysis: A Practitioner's Guide 

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## The effects of improving sleep on mental health (OASIS): a randomised controlled trial with mediation analysis

Daniel Freeman, Bryony Sheaves, Guy M Goodwin, Ly-Mee Yu, Alecia Nickless, PaulJ Harrison, Richard Emsley, Annemarie I Luik, Russell G Foster, Vanashree Wadekar, Christopher Hinds, Andrew Gumley, Ray Jones, Stafford Lightman, Steve Jones, Richard Bentall, Peter Kinderman, Georgina Rowse, Traolach Brugha, Mark Blagrove, Alice M Gregory, Leanne Fleming, Elaine Walklet, Cris Glazebrook, E Bethan Davies, Chris Hollis, Gillian Haddock, Bev John, Mark Coulson, David Fowler, Katherine Pugh, John Cape, Peter Moseley, Gary Brown, Claire Hughes, Marc Obonsawin, Sian Coker, Edward Watkins, Matthias Schwannaver, Kenneth MacMahon, A Niroshan Siriwardena, Colin A Espie

## Summary

Background Sleep difficulties might be a contributory causal factor in the occurrence of mental health problems. If this is true, improving sleep should benefit psychological health. We aimed to determine whether treating insomnia leads to a reduction in paranoia and hallucinations

## WHAT IS MEDIATION ANALYSIS?



Exposure (a) $\longrightarrow$ Mediator $(\mathrm{m}) \longrightarrow$ Outcome $(\mathrm{Y})$

Confounder
Explain mechanism of relationship between exposure and outcome by third variable (mediator)

$$
\text { Exposure }(\mathrm{a}) \longrightarrow \text { Mediator }(\mathrm{m}) \longrightarrow \text { Outcome }(\mathrm{Y})
$$

Cognitive behavioral Therapy Insomnia Paranoia Vs

Confounder
Psychiatric disorder Substance abuse

$$
\text { Exposure }(\mathrm{a}) \longrightarrow \text { Mediator }(\mathrm{m}) \longrightarrow \text { Outcome }(\mathrm{Y})
$$

Cognitive behavioral Therapy
Vs
Usual Care


Insomnia


## Difference Method

(biomedical science)
$E[Y \mid a, c]=\phi_{0}+\phi_{1} a+\phi_{4} c$.
$E[Y \mid a, m, c]=\theta_{0}+\theta_{1} a+\theta_{2} m+\theta_{4} c$.

$$
\begin{gathered}
\mathrm{DE}=\theta_{1} . \\
\mathrm{IE}=\phi_{1}-\theta_{1} .
\end{gathered}
$$

Product Method (social science)
$E[Y \mid a, m, c]=\theta_{0}+\theta_{1} a+\theta_{2} m+\theta_{4} c$.
$E[M \mid a, c]=\beta_{0}+\beta_{1} a+\beta_{2} c$.
$D E=\theta_{1}$.
$\boldsymbol{I E}=\beta_{1} \theta_{2}$.

## Outcome (Y)



Covariate (c): Gender, educational level

|  | Insomnia (SCI-8) |  |  |  | Paranoia (GPTS) |  |  |  | Hallucinations (SPEQ) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unadjusted mean |  | Adjusted difference*$(95 \% \mathrm{Cl}), \mathrm{d} \dagger$ | $p$ value* | Unadjusted mean |  | Adjusted difference* (95\% CI), d $\dagger$ | p value* | Unadjusted mean |  | Adjusted difference* <br> (95\% CI), d $\dagger$ | p value* |
|  | Control | Treatment |  |  | Control | Treatment |  |  | Control | Treatment |  |  |
| Week 3 | $\begin{gathered} 12 \cdot 34 \\ (5 \cdot 85) \end{gathered}$ | $\begin{gathered} 14.96 \\ (5.80) \end{gathered}$ | $\begin{aligned} & 2.62 \text { (2.19 to } 3.06 \text { ), } \\ & 0.61 \end{aligned}$ | <0.0001 | $\begin{gathered} 24.63 \\ (11.82) \end{gathered}$ | $\begin{array}{r} 22.61 \\ (9.89) \end{array}$ | $\begin{aligned} & -1.81(-2.49 \text { to }-1.13), \\ & 0.15 \end{aligned}$ | <0.0001 | $\begin{array}{r} 5.06 \\ (6.89) \end{array}$ | $\begin{array}{r} 4.06 \\ (5.84) \end{array}$ | $\begin{aligned} & -0.79(-1.15 \text { to }-0.42), \\ & 0.12 \end{aligned}$ | <0.0001 |
| Week 10 | $\begin{gathered} 13.31 \\ (6.45) \end{gathered}$ | $\begin{gathered} 18.08 \\ (6.66) \end{gathered}$ | $\begin{aligned} & 4.78(4 \cdot 29 \text { to } 5 \cdot 26), \\ & 1 \cdot 11 \end{aligned}$ | <0.0001 | $\begin{array}{r} 23 \cdot 84 \\ (12 \cdot 16) \end{array}$ | $\begin{aligned} & 21.06 \\ & (9.08) \end{aligned}$ | $\begin{aligned} & -2.22(-2.98 \text { to }-1.45), \\ & 0.19 \end{aligned}$ | <0.0001 | $\begin{gathered} 4.89 \\ (7.24) \end{gathered}$ | $\begin{array}{r} 3 \cdot 12 \\ (5 \cdot 12) \end{array}$ | $\begin{aligned} & -1.58 \text { (-1.98 to -1.18), } \\ & 0.24 \end{aligned}$ | <0.0001 |
| Week 22 | $\begin{aligned} & 14 \cdot 43 \\ & (6 \cdot 71) \end{aligned}$ | $\begin{gathered} 19 \cdot 27 \\ (7 \cdot 13) \end{gathered}$ | $\begin{aligned} & 4.81(4 \cdot 29 \text { to } 5 \cdot 33), \\ & 1.12 \end{aligned}$ | <0.0001 | $\begin{gathered} 23.84 \\ (12.68) \end{gathered}$ | $\begin{array}{r} 20 \cdot 75 \\ (9 \cdot 19) \end{array}$ | $\begin{aligned} & -2.78(-3.60 \text { to }-1 \cdot 96), \\ & 0.24 \end{aligned}$ | <0.0001 | $\begin{array}{r} 4.71 \\ (7 \cdot 43) \end{array}$ | $\begin{array}{r} 2.87 \\ (5.45) \end{array}$ | $\begin{aligned} & -1.56 \text { (-1.99 to -1.14), } \\ & 0.23 \end{aligned}$ | <0.0001 |

Data are mean (SD). At week 3, 1398 participants were in the control group and 1044 participants were in the treatment group. At week 10,1142 participants were in the control group and 733 participants were in the treatment group. At week 22, 971 participants were in the control group and 603 participants were in the treatment group. SCI-8=Sleep Condition Indicator 8 -item version. GPTS=Green et al Paranoid Thought Scales. SPEQ=Specific Psychotic Experiences Questionnaire. *Linear mixed effects model adjusted for gender, student status, week, and interaction of week with randomisation, and including a random effect for student within university. Covariance matrix of within subject measurements was unstructured. $\uparrow d$ is standardised effect size (Cohen's d).

Table 2: Primary outcome results


## CONFOUNDING ASSUMPTION

Assumption 1: Control Exposure -> Outcome<br>Assumption 2: Control Mediator -> Outcome<br>Assumption 3: Control Exposure -> Mediator<br>Assumption 4: No Exposure->Mediator-> Outcome



Relations between exposure $A$, mediator $M$, and outcome $Y$, and confounders.


Figure 2
A mediator-outcome confounder $L$ that is itself affected by the exposure $A$.

## EXPOSURE-MEDIATOR INTERACTION

Exposure (a)


Mediator (m) $\longrightarrow$ Outcome ( Y )


## EXPOSURE-MEDIATOR INTERACTION



## EXPOSURE-MEDIATOR INTERACTION



$$
\begin{aligned}
\mathrm{E}[Y \mid a, m, c] & =\theta_{0}+\theta_{1} a+\theta_{2} m+\theta_{3} \mathrm{am}+\theta_{4} \mathrm{c} \\
\mathrm{E}[M \mid a, c] & =\beta_{0}+\beta_{1} a+\beta_{2} \mathrm{c} . \\
\mathrm{DE} & =\left\{\theta_{1}+\theta_{3}\left(\beta_{0}+\beta_{1} \mathrm{a} *+\beta_{2} c\right)\right\}(a-a *) \\
\mathrm{a} \quad \mathrm{IE} & =\left(\beta_{1} \theta_{2}+\beta_{1} \theta_{3} a\right)(a-a *)
\end{aligned}
$$

Time 3 weeks = a
Time 10 weeks $=\mathrm{a}^{*}$

|  | Insomnia (SCl-8) |  |  |  | Paranoia (GPTS) |  |  |  | Hallucinations (SPEQ) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unadjusted mean |  | Adjusted difference* ( $95 \% \mathrm{Cl}$ ) $\mathrm{d} \dagger$ | p value* | Unadjusted mean |  | Adjusted difference* <br> (95\% CI), d $\dagger$ | p value* | Unadjusted mean |  | Adjusted difference* (95\% CI), d $\dagger$ | p value* |
|  | Control | Treatment |  |  | Control | Treatment |  |  | Control | Treatment |  |  |
| Week 3 | $\begin{aligned} & 12 \cdot 34 \\ & (5 \cdot 85) \end{aligned}$ | $\begin{aligned} & 14.96 \\ & (5.80) \end{aligned}$ | $\begin{aligned} & 2.62(2.19 \text { to } 3.06), \\ & 0.61 \end{aligned}$ | <0.0001 | $\begin{gathered} 24.63 \\ (11.82) \end{gathered}$ | $\begin{gathered} 22.61 \\ (9.89) \end{gathered}$ | $\begin{aligned} & -1 \cdot 81(-2 \cdot 49 \text { to }-1 \cdot 13), \\ & 0 \cdot 15 \end{aligned}$ | <0.0001 | $\begin{array}{r} 5.06 \\ (6.89) \end{array}$ | $\begin{array}{r} 4.06 \\ (5.84) \end{array}$ | $\begin{aligned} & -0.79(-1.15 \text { to }-0.42), ~ \\ & 0.12 \end{aligned}$ | <0.0001 |
| Week 10 | $\begin{aligned} & 13 \cdot 31 \\ & (6.45) \end{aligned}$ | $\begin{gathered} 18.08 \\ (6.66) \end{gathered}$ | $\begin{aligned} & 4.78 \text { ( } 4 \cdot 29 \text { to } 5 \cdot 26 \text { ), } \\ & 1 \cdot 11 \end{aligned}$ | <0.0001 | $\begin{array}{r} 23 \cdot 84 \\ (12 \cdot 16) \end{array}$ | $\begin{gathered} 21.06 \\ (9.08) \end{gathered}$ | $\begin{aligned} & -2.22(-2.98 \text { to }-1.45), \\ & 0.19 \end{aligned}$ | <0.0001 | $\begin{array}{r} 4.89 \\ (7.24) \end{array}$ | $\begin{gathered} 3 \cdot 12 \\ (5 \cdot 12) \end{gathered}$ | $\begin{aligned} & -1 \cdot 58(-1 \cdot 98 \text { to }-1 \cdot 18), \\ & 0.24 \end{aligned}$ | <0.0001 |
| Week 22 | $\begin{aligned} & 14 \cdot 43 \\ & (6.71) \end{aligned}$ | $\begin{gathered} 19 \cdot 27 \\ (7 \cdot 13) \end{gathered}$ | $\begin{aligned} & 4 \cdot 81 \text { (4.29 to } 5 \cdot 33), \end{aligned}$ | <0.0001 | $\begin{gathered} 23.84 \\ (12.68) \end{gathered}$ | $\begin{gathered} 20.75 \\ (9 \cdot 19) \end{gathered}$ | $\begin{aligned} & -2.78 \text { (-3.60 to }-1.96), \\ & 0.24 \end{aligned}$ | <0.0001 | $\begin{array}{r} 4.71 \\ (7.43) \end{array}$ | $\begin{array}{r} 2.87 \\ (5.45) \end{array}$ | $\begin{aligned} & -1.56(-1.99 \text { to }-1.14) \text {, } \\ & 0.23 \end{aligned}$ | <0.0001 |

Data are mean (SD). At week 3, 1398 participants were in the control group and 1044 participants were in the treatment group. At week 10, 1142 participants were in the control group and 733 participants were in the treatment group. At week 22, 971 participants were in the control group and 603 participants were in the treatment group. SCI-8=Sleep Condition Indicator 8-item version. GPTS=Green et al Paranoid Thought Scales. SPEQ=Specific Psychotic Experiences Questionnaire. *Linear mixed effects model adjusted for gender, student status, week, and interaction of week with randomisation, and including a random effect for student within university. Covariance matrix of within subject measurements was unstructured. $\dagger \mathrm{d}$ is standardised effect size (Cohen's d ).

## Table 2: Primary outcome results

## BINARY OUTCOMES AND MEDIATORS

$$
\begin{aligned}
& \operatorname{logit\{ P(Y=1|a,m,c)\} =\theta _{0}+\theta _{1}a+\theta _{2}m+\theta _{3}am+\theta _{4}c} \\
& E[M \mid a, c]=\beta_{0}+\beta_{1} a+\beta_{2} c \\
& \log \left\{O R^{D E}\right\} \sim=\left\{\theta_{1}+\theta_{3}\left(\beta_{0}+\beta_{1} a+\beta_{2} c+\theta_{2} \sigma^{2}\right)\right\}(a-a *)+0.5 \theta_{3}^{2} \sigma^{2}\left(a^{2}-a *^{2}\right) \\
& \log \left\{O R^{1 E}\right\} \sim=\left(\theta_{2} \beta_{1}+\theta_{3} \beta_{1} a\right)(a-a *)
\end{aligned}
$$

$$
\begin{aligned}
D E & =\theta_{1}\left(a-a^{*}\right)+\theta_{3}\left(a-a^{*}\right) \frac{\exp \left(\beta_{0}+\beta_{1} a^{*}+\beta_{2}^{\prime} c\right)}{1+\exp \left(\beta_{0}+\beta_{1} a^{*}+\beta_{2}^{\prime} c\right)} \\
I E & =\left(\theta_{2}+\theta_{3} a\right)\left\{\frac{\exp \left(\beta_{0}+\beta_{1} a+\beta_{2}^{\prime} c\right)}{1+\exp \left(\beta_{0}+\beta_{1} a+\beta_{2}^{\prime} c\right)}-\frac{\exp \left(\beta_{0}+\beta_{1} a^{*}+\beta_{2}^{\prime} c\right)}{1+\exp \left(\beta_{0}+\beta_{1} a^{*}+\beta_{2}^{\prime} c\right)}\right\} .
\end{aligned}
$$

## CONCLUSION

Mediation Analysis can be used to assess the relative magnitude of pathway and mechanisms by which exposure may affect the outcome

